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VOLT-AMPERE CHARACTERISTICS OF A LONGITUDINALLY BLOWN SELF-ADJUSTING ARC IN PLASMA GENERATORS WITH A ROD CATHODE

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Using the methods of regression analysis, consideration is given to the influence of various processes on the volt-ampere characteristics (VAC) of an arc discharge in a plasma generator with a hot rod cathode and a cold cylindrical anode with an arc length that is self-adjusting due to the mechanism of shunting. It is shown that heat-transfer processes play a dominant role in VAC formation.

Introduction. Modeling of electric-arc discharges is a very difficult problem by virtue of a complicated combination of interrelated processes, high temperatures, and a considerable difference in the temperature dependence of the plasma properties for various media. Therefore, in modeling electric discharges use is made of approximate methods with consideration of certain processes that exert the main influence on the discharge characteristics. But even in this simplified approach mathematical modeling can be accomplished only for stable discharges of simple geometry. In the overwhelming majority of cases, however, various electric-arc apparatuses deal with unstable discharges of complicated configuration. For these discharges it is necessary to employ physical simulation.

Nevertheless, physical simulation also encounters considerable difficulties. Choosing the dominant processes is a complicated task because of their dependence on the discharge conditions, which leads to a decrease in the accuracy of the characteristics obtained as the range of the parameters covered by a unified expression broadens. Moreover, the very wide limits of variation of the temperature over an arc-column cross section make it difficult to choose scale values of the plasma properties required for unified characteristics applied to different working media.

An arc discharge in a cylindrical channel in the case of longitudinal eddy blowing of it by a heated gas is highly unstable if the discharge channel simultaneously serves as one of the electrodes of the heater. In this case, periodic electric breakdown between the arc column and the electrode wall and column elongation in the intervals between breakdowns occur. This process of "shunting" exerts a strong influence on the volt-ampere characteristic of the discharge and complicates its modeling since an additional mechanism of electric breakdown is involved that, in turn, is interrelated with the basic processes of energy transfer. Nevertheless, as experience shows, physical simulation can be applied even to such complicated phenomena as a longitudinally blown arc with its length self-adjusting due to shunting.

Unlike previous attempts at physical simulation of longitudinally blown arc discharges, in the present work use is made of the methods of modern regression analysis and computational programs developed for personal computers. This has made it possible to perform a careful selection of the dominant processes and to refine their characteristics. In extention of [1], in the present work consideration is given to the influence of the kind of heated gas and the anode diameter.

1. Experimental Data and Procedure of Their Processing. An analysis of the volt-ampere characteristics of a longitudinally blown arc is made using data of our experiments and literature data [2-6]. A schematic of the discharge device is shown in Fig. 1. Arc 1 burned between rod cathode 2 and cylindrical anode 3 in heated gas 4

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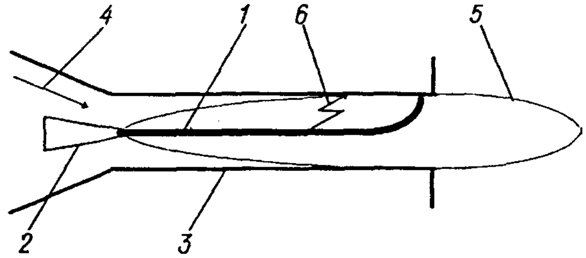

Fig. 1. Schematic of a plasma generator with a self-adjusting arc length: 1) arc, 2) rod cathode, 3) cylindrical anode, 4) heated gas, 5) plasma jet, 6) arc shunting.

supplied on the side of the cathode with swirling. The anode and the cathode were cooled with running water. The heated gas emanated in the form of plasma jet 5. The arc was lengthened under action of the heated-gas flow. Here, the potential difference between the arc and the electrode wall increased, thus causing breakdown of the near-wall gas layer in some region 6. Investigations were made for discharges blown with air, argon, and hydrogen.

The investigated parameters for an air discharge ranged as follows: gas flow rate $G = 0.62-31.5$ g/sec; current strength $I = 10-250$ A; anode diameter $d = 5$, 9.52, 10, and 20 mm ([2, 3], our experiments). Correspondingly, for an argon discharge: $G = 0.44-2.5$ g/sec; $I = 24.2-167.8$ A; $d = 4$, 8, and 16 mm ([4, 5], our experiments); for a hydrogen discharge: $G = 0.75-1.5$ g/sec; $I = 350-900$ A; $d = 10$ mm [6].

For an analysis of the dominant mechanisms exerting the main influence on the discharge characteristics the VACs of the arcs were presented in the form of the generalized power-law expression

$$\pi_{\text{dep}} = C \prod_i \pi_i^{\alpha_i}. \tag{1}$$

Relation (1) was linearized using natural logarithms. The variables in this formula are dimensionless similarity numbers. As the dependent variable, the generalized discharge resistance $\pi_{\text{dep}} = Ud\sigma_0/I$ was adopted.

The initial set of assumed important independent variables included the following similarity numbers: $\pi_{\text{cond}} = \sigma_0\lambda_0T_0d^2/I^2$ is the conductive energy transfer of the Joule dissipation; $\pi_{\text{conv}} = Gd\sigma_0h_0/I^2$ is the convective energy transfer of the Joule dissipation; $\pi_{\text{rad}} = \sigma_0Q_0d^4/I^2$ is the radiative energy transfer of the Joule dissipation; $\pi_{\text{turb}} = \sigma_0\rho_0h^{1.5}d^3/I^2$ is the turbulent energy transfer of the Joule dissipation; $\text{Re}_{\text{hot}} = G/\eta_{\text{b.hot}}d$ is the Reynolds number for a discharge; $\text{Re}_{\text{cold}} = G/\eta_{\text{b.cold}}d$ is the Reynolds number for a blowing cold gas.

For selection of the dominant similarity numbers that must describe the discharge VACs, multiple linear regression programs were used. In doing so, the procedure of "forward" step-by-step testing was adopted [7]. The program successively selected independent variables (regressors) providing the highest value of the determinant R^2, which represented the share of the sum of the squares of the deviation of the dependent variable from the overall mean covered by the regression. On introduction of several regressors into the model, their relative effect was determined more precisely. The selection ceased if upon introducing the next regressor the Fisher number F turned out to be lower than the threshold characterizing the F-distribution at the prescribed confidence coefficient. Usually F = 4.0 was prescribed, which in a wide range of the number of degrees of freedom was close to the F-distribution at the 95% probability that F exceeds the tabular value (the 5% level of probability).

Since the processes in an arc are interrelated, the independent variables representing them are highly correlated (multicollinearity), which can be seen, for instance, in the correlation matrix for a hydrogen discharge (Table 1). This hinders use of the method of least squares for optimization of the choice of regression coefficients. Refinement of the relative role of individual independent variables under multicollinearity conditions is achieved by a certain shift of the regression coefficients. Therefore, the adopted multiple linear regression included a program of "ridge" regression ensuring a shift of the coefficients. Usually the regularization coefficient $\alpha = 0.1$ was used.

The representative properties constituting the similarity numbers were determined by a method [8] that takes into account the temperature dependence of the plasma properties [9].

TABLE 1. Correlation Matrix of the VAC of a Hydrogen Arc

Criteria	π_{conv}	π_{rad}	π_{turb}	π_{cond}	$\pi_{Re.hot}$	$\pi_{Re.cold}$	π_{dep}
π_{conv}	1.00	0.99	0.99	1.00	−0.83	−0.83	1.00
π_{rad}	0.99	1.00	1.00	0.99	−0.85	−0.87	0.99
π_{turb}	0.99	1.00	1.00	1.00	−0.87	−0.87	1.00
π_{cond}	1.00	0.99	1.00	1.00	−0.86	−0.86	1.00
$\pi_{Re.hot}$	−0.83	−0.87	−0.87	−0.86	1.00	1.00	−0.84
$\pi_{Re.cold}$	−0.83	−0.87	−0.87	−0.86	1.00	1.00	−0.84
π_{dep}	1.00	0.99	1.00	1.00	−0.84	−0.84	1.00

TABLE 2. Regression Parameters of Generalized VACs of Different Gases

Heated gas	Variables	β	Error of β	B	Error of B	Student quantile	Probability level
Argon	c	–	–	0.532	0.199	2.67	0.012
$R^2=0.819$	π_{rad}	0.531	0.153	0.233	0.067	3.47	0.002
$SE=0.338$	π_{turb}	0.362	0.153	0.170	0.072	2.37	0.025
$F_{(2.28)}=63.2$							
Air	c	–	–	−1.358	0.317	−4.29	0.000
$R^2=0.956$	π_{conv}	0.250	0.048	0.164	0.032	5.19	0.000
$SE=0.224$	π_{cond}	0.246	0.041	0.227	0.038	6.04	0.000
$F_{(6.129)}=468$	π_{turb}	0.197	0.048	0.158	0.039	4.07	0.000
	π_{rad}	0.148	0.038	0.097	0.025	3.94	0.000
	Re_{hot}	0.143	0.044	0.176	0.054	3.25	0.001
	Re_{cold}	0.143	0.044	0.176	0.054	3.25	0.001
Hydrogen	c	–	–	1.780	0.333	5.34	0.000
$R^2=0.974$	π_{conv}	0.252	0.061	0.192	0.047	4.11	0.000
$SE=0.415$	π_{cond}	0.236	0.062	0.166	0.044	3.79	0.000
$F_{(0.49)}=457$	π_{rad}	0.246	0.060	0.156	0.038	4.09	0.000
	π_{turb}	0.242	0.063	0.162	0.042	3.84	0.000
Air and	c	–	–	−0.406	0.298	−1.36	0.175
hydrogen	π_{conv}	0.310	0.037	0.224	0.027	8.36	0.000
$R^2=0.966$	π_{cond}	0.277	0.037	0.208	0.028	7.48	0.000
$SE=0.330$	π_{rad}	0.055	0.022	0.021	0.008	2.55	0.012
$F_{(5.184)}=1040$	π_{turb}	0.324	0.035	0.230	0.025	9.14	0.000
	Re_{hot}	0.109	0.017	0.251	0.040	6.33	0.000

2. Choice of Variables. Data on the selection of the dominant independent variables are provided in Table 2, where generalized arguments are shown, the influence of which is beyond the limits of the errors. For VACs of an arc blown by individual gases and for combined VACs of air and hydrogen discharges Table 2 also gives the values of the determinant R^2, the regression ratio of Fisher dispersions F, and standard errors of the regression SE.

The relative role of the individual independent variables is reflected by the value of the standardized coefficients β. They show a considerable difference in the influence of the plasma properties of different gases on the VACs. The most pronounced influence on the VACs in the case of air and hydrogen is exerted by convection, and in the case of argon, by radiation. In the molecular gases the conductive energy transfer also plays a consid-

TABLE 3. Comparison of the Exponents of Empirical and Generalized VACs

Gas	Types of regression								
	empirical			generalized ordinary			generalized ridge		
	α	β	γ	α	β	γ	α	β	γ
Argon	0.198	0.272	0.357	0.192	0.263	0.353	0.192	–	0.442
Air	−0.338	0.530	0.132	−0.354	0.521	0.145	−0.292	0.512	0.128
Hydrogen	−0.451	0.685	0.383	−0.444	0.703	0.463	−0.352	0.192	0.634

erable role, while the turbulent transfer is appreciable for a discharge in all the gases. The Reynolds number is manifested only in the air arc and its effect is weaker as compared to each of the heat-transfer processes. The values of β in Table 2 do not agree with the data for the electric-field intensity in the arc column, where the turbulence and the conduction play an important role.

The considerable difference in the plasma properties leads to different VAC slopes for the individual gases, as follows from the data on the natural coefficients of multiple regression (Table 2). As is seen, in hydrogen and air the VACs have a descending slope (the arc voltage decreases with increase in the current strength) since the sum of the coefficients for the current-containing variables π_{conv}, π_{cond}, π_{turb}, and π_{rad} is larger than 0.5. Here, in hydrogen the voltage decreases more sharply than in air. In argon, the VAC increases since the sum of the coefficients is smaller than 0.5. The difference in the coefficients for air and hydrogen is not important (0.642 and 0.676) and for these gases a unified VAC can be obtained (Table 2). Addition of argon to them is not reasonable since the blown arc has an ascending VAC despite the discharge shunting.

3. Influence of the Variables Omitted in the Initial Set of Numbers. The influence of the omitted variables on the arc characteristics can be found by comparing the exponents of the empirical ($U = CI^{\alpha}G^{\beta}d^{\gamma}$) and generalized VACs. Results of this comparison are given in Table 3. It is seen that the ridge regression as compared to the empirical one yields considerably different values of the exponents β and γ. This indicates that some important number is not taken into consideration. The omitted variable must presumably reflect the influence of gas-flow swirling on the electric breakdown between the arc and the electrode wall. This assumption is supported by the weak influence of the gas flow rate on the VACs of the argon arc because the shunting process is stimulated mainly by radiation. Consequently, the mechanism of an electric breakdown depends on the heat-transfer processes, and the arc shunting is of a thermal nature. This mode of breakdown is confirmed by the weak influence of the convective energy transfer on the $E-I$ characteristic of the arc (E is the electric-field intensity) burning in air and hydrogen. At the same time the VAC of discharges in these gases depends strongly on convection. Therefore the neglected number must be independent of the gas heating.

4. Generalized VACs. The data presented in Table 2 make it possible to build generalized VACs for the individual gases and a unified VAC for air and hydrogen. The natural coefficients B (Table 2) represent the exponents of the corresponding generalized variables in expression (1), while the constant term c in the table corresponds to the natural logarithm of the coefficient. For instance, the generalized VAC for air is of the form

$$\frac{Ud\sigma_0}{I} = 0.257\left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{0.164}\left(\frac{\sigma_0\rho_0 h_0^{1.5}d^3}{I^2}\right)^{0.158}\left(\frac{\sigma_0\lambda_0 T_0 d^2}{I^2}\right)^{0.227}\left(\frac{\sigma_0 Q_0 d^4}{I^2}\right)^{0.097}\left(\frac{G}{\eta_{\text{hot}}d}\right)^{0.176}\left(\frac{G}{\eta_{\text{cold}}d}\right)^{0.176}. \quad (2)$$

Despite the fact that such expressions reflect the relative role of individual processes, they are cumbersome and inconvenient for practical calculations. More concise formulas are obtained by the method of ordinary (not of the ridge type) multiple regression, in which a number of strongly correlated variables are expressed in terms of the one manifesting the highest effect. Below we give generalized VACs obtained by this method:

for a discharge in air

$$\frac{Ud\sigma_0}{I} = 0.934\left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{0.521}\left(\frac{\sigma_0 Q_0 d^4}{I^2}\right)^{0.156}, \quad (3)$$

TABLE 4. Separation of Generalized VACs According to Data of Different Authors and with Respect to the Kind of Working Gas

Character of VAC separation	F_{inad}	$F_{5\%}$	F_{reg}
With respect to different constructions:			
argon	2.74	2.99	77
hydrogen	3.78	2.79	35,622
air	52.30	2.17	2080
With respect to the kind of gas	62.25	1.98	3347

for a discharge in hydrogen

$$\frac{Ud\sigma_0}{I} = 0.0018 \left(\frac{\sigma_0 \rho_0 h_0^{1.5} d^3}{I^2}\right)^{0.722} \left(\frac{G}{\eta_{hot} d}\right)^{0.685} \left(\frac{\sigma_0 \lambda_0 T_0 d^2}{I^2}\right)^{0.109}, \tag{4}$$

the unified VAC for air and hydrogen

$$\frac{Ud\sigma_0}{I} = 0.231 \left(\frac{Gd\sigma_0 h_0}{I^2}\right)^{0.349} \left(\frac{\sigma_0 \rho_0 h_0^{1.5} d^3}{I^2}\right)^{0.364} \left(\frac{G}{\eta_{hot} d}\right)^{0.155}, \tag{5}$$

for a discharge in argon

$$\frac{Ud\sigma_0}{I} = 0.407 \left(\frac{\sigma_0 Q_0 d^4}{I^2}\right)^{0.404} \left(\frac{G}{\eta_{hot} d}\right)^{0.263}. \tag{6}$$

The standard errors of the expressions obtained are smaller than in the case of use of the ridge regression. Uniting the VACs of discharges for air and hydrogen by the unified expression did not increase the error as compared to the VAC for air.

5. Separation of the VACs. In connection with the approximate character of the similarity of the generalized VACs some separation can take place in the individual dimensional variables entering the similarity numbers and in different working media. This is not so substantial that the expression is inadequate, but it can be evaluated quantitatively by the Fisher inadequacy parameter F_{inad}. This inadequacy is calculated by the ratio of the corresponding dispersion S^2_{inad} to the pure-error dispersion $S^2_{p.e}$, $F_{inad} = S^2_{inad}/S^2_{p.e}$. Neglecting the inadequacy of the expressions for the VAC calculated for each of certain parameters (e.g., the electrode diameter, the flow rate or kind of gas), it can be considered that the entire remaining scatter of these VACs relative to the regression is attributable to random factors and corresponds to the pure error $S_{p.e} = \sum_i SS_i$. Then the difference between the sum of the squares of the deviations in the unified VAC and the sum of the squares of the pure error amounts to the sum of the squares of the inadequacy $SS_{inad} = SS_{gen} - SS_{p.e}$. The same method is used to calculate the number of degrees of freedom of the pure error and the inadequacy and to determine $S^2_{p.e}$ and S^2_{inad}.

Table 4 provides values of F_{inad} characterizing the separations of the generalized VACs with respect to different gases and different constructions (i.e., according to the data of different authors) for individual gases. The values of F_{inad} obtained are compared to the corresponding tabular values. It is seen that in the case of hydrogen and argon no separation in relation to the constructions is found. This can be explained by the large scatter of the results of observations in individual VACs. In the case of air, where the accuracy of individual VACs is higher, marked separation in the constructions is observed. The separation in different gases is approximately of the same magnitude as in the constructions. However, the Fisher parameter for separation is low as compared to that for regression.

Conclusion. The regression analysis of experimental data on the VACs of arc discharges in air, argon, and hydrogen at atmospheric pressure has revealed that the heat-transfer processes exert the main influence on formation of this characteristic while the Reynolds number, usually adopted for VAC generalization, turns out to be insignificant. Moreover, a thermal character for the electric breakdown between an arc and an electrode is indicated. It is also shown that consideration of the swirling effect of a heated gas can improve generalized VACs of plasma generators with a self-adjusting arc length.

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NOTATION

d, anode diameter; G, flow rate of the heated gas; h, enthalpy; I, current; Q, plasma emissivity; T, temperature; U, arc voltage; α, exponent; η, coefficient of dynamic viscosity; λ, thermal conductivity; π, similarity number; ρ, density; σ, electrical conductivity.

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